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Design of Robust Control for Block Nonlinear Systems by Lyapunov Functions Method

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Abstract. This paper is devoted to robust control design for block multilinked nonlinear dynamical systems. Transformation of the block system to the single block system is proposed. For the considered block systems function of Lyapunov is designed. It is proved if the number of controls is equal to or more than the number of state variables of the block, then in the given area the closed-loop system conditions of stability followed controllability conditions. Control design accounts limitations of controls and state variables. Modeling results for nonlinear objects control systems are presented.

Introduction

Modern control design methods are based on the canonical forms of mathematical models. For the linear systems Controllable-Canonical Form is well known [1]. But there is no conventional canonical form for the nonlinear systems. Therefore for the nonlinear systems different canonical forms are applied. There are controllable Jordan form [2], normal canonical form [3], block form [4, 5], cascade form [6]. In this paper robust control design problem on base of controllable Jordan form is considered. To solve the problem position and path control method is applied [4, 7].

Control For The Single Block Object

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Assume the mathematical model of controlled object is

$$\dot{x} = f(x) + Bu \tag{1}$$

Herein x is state variables vector $n\times 1$, u is control vector $m\times 1$, $f(x) = \left(f_i(x)\right)^T$, $i = \overline{1,n}$, is functional vector, $f_i(x)$ are function satisfied the existence and uniqueness conditions, $B = (b_i)$, $i = \overline{1,n}$, is matrix $n\times m$, b_i are row vectors.

Assume that controls are limited:

$$\left|u_j\right| \le u_j^{\max}, \quad j = \overline{1,m} \tag{2}$$

Herein $u_j^{\max}$ are positive constants.

It is necessary find control vector $u = \begin{pmatrix} u_1 & u_2 & \dots & u_m \end{pmatrix}^T$ as function of system (1) state variables with limitations (2). The control vector must transit object (1) from an arbitrary initial state $x_0 \ne 0$ to a given final state $x_k = 0$. Stability of the closed-loop system must be ensured.

Let us consider a single block object.

Now we introduce the following definition 1.

Definition 1. Object (1) consists of a single block if inequalities (3) are satisfied.

$$\left\|b_i\right\| \ne 0, \quad i = \overline{1,n} \tag{3}$$

Control design is based on the following theorem 1.

Theorem 1. For object (1) – (3) robust control is

$$u = -U^{\max} \tanh\left(QB^T(x)x\right) \quad (4)$$

Herein $U^{\max} = diag\left(u_1^{\max}, u_2^{\max}, u_m^{\max}\right)$ is a diagonal matrix, tanh is hyperbolic tangent function, $Q = diag(q_1, q_2, ..., q_m)$ is a diagonal matrix of the control parameters.

If condition (5) and (6) are satisfied

$$\left|\left(b_i, u^{\max}\right)\right| > \left|f_i(x)\right|,\ i = \overline{1,n} \quad (5)$$

$$BU^{\max}QB^T > \alpha,\ \left|f(x)\right| < \left|(\alpha, x)\right|,\ i = \overline{1,n} \quad (6)$$

then control system (1) – (4) is asymptotically stable.

Herein operation $|f(x)|$ is absolute value of every element of $f(x)$, and $u^{\max} = \left(u_1^{\max}, u_2^{\max}, u_m^{\max}\right)$ is a vector .

Lyapunov function of the system (1) – (4) is

$$V = 0.5x^T x \quad (7)$$

Proof of Theorem 1. From differentiating (7) we obtain

$$\dot{V} = x^T\dot{x} = x^T\left(f(x) - BU^{\max}\tanh\left(QB^T x\right)\right) \quad (8)$$

Consider areas noted by 1 and 2 and shown on Fig. 1.

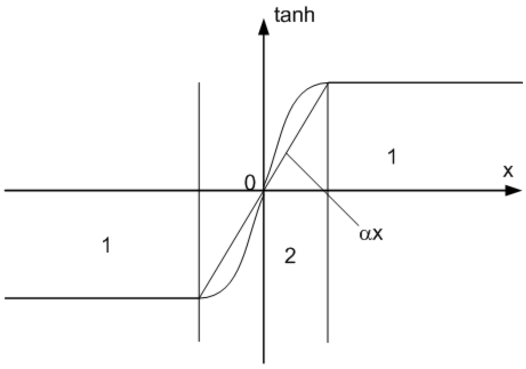


Fig.1 – To proof the systen stabilty

In area 2 we have

$$\tanh\left(QB^T x\right) \approx QB^T x \quad (9)$$

From (8) and (9) we obtain

$$\begin{aligned}\dot{V} &= x^T\left(f(x) - BU^{\max}QB^T x\right) > \\ &> x^T\left(\alpha x - BU^{\max}QB^T x\right) = \\ &= x^T\left(\alpha - BU^{\max}QB^T\right)x\end{aligned} \quad (10)$$

If inequalities (6) are satisfied, then derivative (10) is negative.

In area 1 we have

$$\tanh\left(QB^T x\right) \approx sign\left(QB^T x\right) \quad (11)$$

From (10) and (7) we obtain

$$\dot{V} = x^T\left(f(x) - BU^{\max}sign\left(QB^T x\right)\right) \quad (12)$$

If inequalities (5) are satisfied, then derivative (12) is negative.

The proof is complete. If gain matrix Q ensures stability margin shown on Fig. 1, then the proof is strict.

From theorem 1 it follows that control (4) ensures stability of the closed-loop system (1) – (4) if (5), (6) are satisfied. Inequalities (5) are controllability conditions of Pyatnickiy [9]. Inequalities (6) are sector limits for functional vector $f(x)$. The first inequality (6) is expression for matrix Q calculation.

If the number of controls is above or equal to the number of state variables, then the next Theorem 2 is true.

Theorem 2. If for object (2) the number of controls is above or equal to the number of state variables

$$m \geq n\,, \tag{13}$$

and inequalities (5) are satisfied, then object (1) is controllable.

From (5) we obtain

$$\left|\left(b_i, u^{\max}\right)\right| = c_i > \left|f_i(x)\right|,\ i = \overline{1,n} \tag{14}$$

Herein c_i are positive numbers.

From (14) we have

$$\left|Bu^{\max}\right| = c \tag{15}$$

Herein $c = (c_1\ c_2\ \dots\ c_n)$ is a vector of positive constant.

From (15) we obtain

$$Bu^{\max} = \tilde{c} \tag{16}$$

Herein the signs of vector $\tilde{c}$ elements can be different from the signs of vector c elements.

System (16) has a nontrivial solution if condition (17) is satisfied:

$$rang(B) = n \tag{17}$$

If (19) is satisfied, then Kalman controllability criteria is performed.

Control For The Block Object

If object (1) consists of few blocks, then the object is described by Definition 2.

Definition 2. If object (1) consists of *k* blocks, then the object mathematical model is

$$\dot{x}^i = f^i\left(x^1,\dots,x^{i+1}\right),\ \dot{x}^k = f^k(x) + Bu\,, \tag{18}$$

Herein $i = \overline{1,k-1}$, $x^i = \left(x_1^i\ \ x_2^i\ \ \dots\ \ x_l^i\right)^T$ are vectors $l \times 1$, $x = \left(x^1, x^2, \dots, x^k\right)^T$, $f(x) = \left(f^1\left(x^1\right),\dots,f^k\left(x^k\right)\right)^T$, $f^i\left(x^1,\dots,x^{i+1}\right) = \left(f_1^i\left(x^1,\dots,x^{i+1}\right)\ \dots\ f_l^i\left(x^1,\dots,x^{i+1}\right)\right)^T$, $\left|\dfrac{\partial f^i\left(x^1,\dots,x^{i+1}\right)}{\partial x^{i+1}}\right| > \varepsilon^i \neq 0$,

$B = \left(b_{ij}\right)$ is matrix $l \times m$, *k* is number of blocks, $n = l \times k$.

In this paper sizes of object (18) blocks are same. In general case sizes of object (18) blocks can be different.

Block object (18) is controllable Jordan form system [2]. If the number of block is more than two $k \geq 2$, then control vector u actuates immediately at vector x^k. Vector x^k actuates at vector x^{k-1}. Therefore vector x^k is fictive control for vector x^{k-1}.

Let us introduce the following transformation:

$$\psi^k = x^k\,,\ \psi^i = h^i x^i + \psi^k\,,\ i = \overline{1,k-1}\,, \tag{19}$$

Herein h^i are weight coefficients. Transformation (19) allows to introduce the Theorem 3.

Theorem 3. If limitations (2) are satisfied for system (18), then robust control is

$$u = -U^{\max}\tanh\left(QB^T\sum_{i=1}^{k}\psi^i\right), \tag{20}$$

If conditions (21) are satisfied

$$\left|\left(kb_j, u^{\max}\right)\right| > \left|\left(\sum_{i=1}^{k} h^i f^i + f^k\right)_j\right|,\ \overline{j = 1,n}$$

$$\left|f^i(x)\right| < \left|\left(\alpha^i, x\right)\right|,\ BU^{\max}QB^T > \alpha^{\max}\,, \tag{21}$$

$$\alpha^{\max} = \max\alpha^i\,,\ i = \overline{1,k}\,,$$

then the closed-loop system (18) – (20) is asymptotically stable. Function of Lyapunov of system (18) – (20) is

$$V = \frac{1}{2}\sum_{i=1}^{k}\left(\psi^i\right)^T\psi^i\,, \tag{22}$$

The proof is same of Theorem 1.

Transformation (19) allows to present system (18) like the single block system. A system of arbitrary form for k=2 can be presented as system (18) by corresponding designation of state variables.

Limitation of State Variables

Let us consider limitations of state variables of the block object (2), (18). Assume that the limitations are

$$\left|x_j^i\right| \le x_j^{\max},\ i=\overline{1,k-1},\ j=\overline{1,l}, \tag{23}$$

For the block object (2), (18), (23) Theorem 4 is true.

Theorem 4. If for system (2), (18), (23) functions $f^i\left(x^1,...,x^{i+1}\right)$ are limited by inequalities (24)

$$\left|f^i\right| < \left|\left(\alpha^i, x^i\right)\right|,\ \left|f^i\right| < \left|x_{\max}^{i+1} b^i\right|,\ \left|f^k\right| < \left|U_{\max} b^k\right|, \tag{24}$$

then robust control is

$$u = -U^{\max} \tanh\left(Q^k B^k \psi^k\right), \tag{25}$$

$$\psi^1 = x^1, \tag{26}$$

$$\psi^i = x^i + x^{i\max} \tanh\left(Q^i B^i \psi^{i-1}\right),\ i=\overline{2,k}, \tag{27}$$

Herein vectors are calculated from inequalities (28), (29)

$$B^{1T} B^1 Q^1 > \alpha^1, \tag{28}$$

$$D^{iT} D^i Q^i > \alpha^i + B^{i-1T} B^{i-1} Q^{i-1},\ i=\overline{2,k}, \tag{29}$$

Control (25) ensures asymptotical stability of system (2), (18), (23), (25). Function of Lyapunov is

$$V = 0.5 \psi^{kT} \psi^k, \tag{30}$$

The proof is same of Theorem 1.

Inequalities (28), (29) are conditions of asymptotical stability of the closed-loop system (2), (18), (23) – (27).

Example 1

Equations of the controlled object are

$$\dot{x}_1 = x_2,\ \dot{x}_2 = u,\ |u| \le U_{\max}, \tag{31}$$

Designed by the developed method robust control for object (31) is

$$u = -U_{\max} \tanh\left(q^2\left(x_2 + x_2^{\max} \tanh\left(q^1 x_1\right)\right)\right), \tag{32}$$

Control (32) is close to bang-bang control [9, 10]. Therefore it is interesting compare the robust control (32) with time-optimal control. The closed time-optimal control for object (31) is [10]

$$u = -U_{\max} sign\left(x_1 + 0.5 x_2^2 sign\left(x_2\right)\right), \tag{33}$$

Herein $sign$ is “*signum*” function.

Modeling results of both the robust system (31), (32) as well as the time-optimal system (31), (33) are presented on fig. 2. Parameters of controls are: $U_{\max} = 2$, $x_2^{\max} = 0.5$, $q^1 = 5,\ q^2 = 10$.

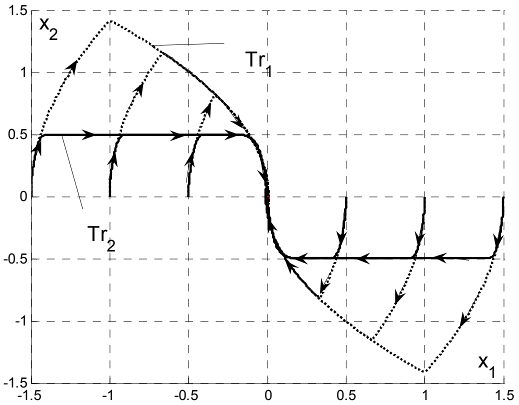


Fig.2 – Phase trajectories of the robust and optimal systems

Tr_1 is the time optimal control system path. Tr_2 is the robust control system path. From fig. 2 we can see that the optimal and robust controls are same in the limited area. To calculate this area it is necessary product the controls (32) and (33). Area of positive sign of product is

$$|x_2| < x_2^{\max}, \ x_1 > 0.5x_2^2, \tag{34}$$

Example 2

Consider mathematical model of vehicle [11]

$$\dot{y} = Rx, \ M\dot{x} = F_u + F_d, \tag{35}$$

Herein x is a speed vector of the aircraft, y is a coordinate vector of the aircraft; F_u is a control vector; F_d is a nonlinear functional vector; M is matrix of masses and moments of inertia; R is functional matrix of kinematical connections. Model (35) is described detailed for airships in [12, 13, 14], for underwater vehicles in [7, 15], and for helicopters in [16].

The aim of this example is to design the vector F_u such that vehicle (35) is stable in an area Ω of the undisturbed motion x^0, y^0.

From transformation (19) we obtain

$$x^1 = x, \ x^2 = x + Hy, \tag{36}$$

Herein H is a nonsingular matrix.

Let the function of Lyapunov be given by

$$V = 0.5\left(x^2\right)^T\left(x^2\right), \tag{37}$$

Differentiating (37) in time we get

$$\dot{V} = \left(x^2\right)^T \dot{x}^2 = \left(x^2\right)^T\left(M^{-1}\left(F_u + F_d\right) + HRx^1\right). \tag{38}$$

Therefore robust control is

$$F_u = -F_u^{\max}\tanh\left(QM^{-1T}\left(x + Hy\right)\right), \tag{39}$$

Herein $F_u^{\max}$ is vector of the control bounds.

It is clear that (38) is a negative definite function if

$$M^{-1}F_u^{\max} > \left|M^{-1}F_d + HRx^1\right|, \tag{40}$$

There are modeling results of system (35), (39) on fig 3. The purpose of the control system is movement along the straight line with speed about 5 m/s.

On fig. 3 V_{x1}, V_{y1}, and V_{z1} are airship speeds in the closed loop system (35), (39). Function F_d is

$$F_d = -0.5c\rho V^2, \tag{41}$$

Herein c is uncertain parameter, ρ is constant, V is air speed of the airship. The uncertain parameter is

$$c = c^0\left(1 + \sin(\omega t)\right)V, \tag{42}$$

Herein c^0, ω are constant parameters.

On fig. 3 areas of inequality (40) failure are marked by rectangles.

On fig. 3 V_{x2}, V_{y2}, and V_{z2} are airship speeds in indirect adaptive control system [17] with constant disturbance model:

$$c = c^0V. \tag{43}$$

We can see from fig. 3 that the robust control system loose the given reference only in the non-controllable area. The adaptive control system operates with error about 12 %.

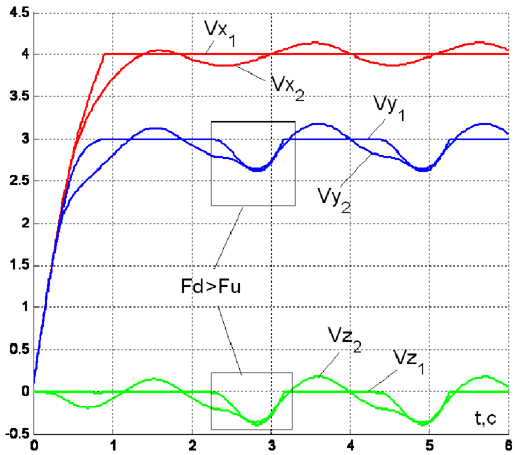


Fig.3 – Phase trajectories of the robust and optimal systems

Experimantal Results

Results of research are implemented in the wheeled mobile robot “Skif”, shown in fig. 4.

Fig.4 – Mobile robot “Skif”

Mathematical model of the mobile robot is

$$\dot{x}_1 = \omega_l a_{11}(\varphi) + \omega_r a_{12}(\varphi),$$
$$\dot{x}_2 = \omega_l a_{21}(\varphi) + \omega_r a_{22}(\varphi), \quad (44)$$
$$\dot{\varphi} = r(\omega_r - \omega_l)/(2b),$$

$$\dot{\omega}_l = -d_1\omega_l + b_{11}u_1 + b_{12}u_2,$$
$$\dot{\omega}_r = -d_2\omega_r + b_{21}u_1 + b_{22}u_2, \quad (45)$$

Herein x_1, x_2 are external coordinates of the robot, φ is angle of orientation of the robot; ω_l, ω_r are the wheel rotation speeds, r is the wheel radius, a is a kinematic parameter, d_i, b_{ij} are constants; u_1, u_2 are controls.

Functions $\alpha_{ij}(\varphi)$ are:

$$\alpha_{11} = 0.5r(\cos\varphi + a\sin\varphi),$$
$$\alpha_{12} = 0.5r(\cos\varphi - a\sin\varphi),$$
$$\alpha_{21} = 0.5r(\sin\varphi - a\cos\varphi), \quad (46)$$
$$\alpha_{22} = 0.5r(\sin\varphi + a\cos\varphi).$$

Let bounds be given by

$$|\omega_l| \le \omega_{\max}, |\omega_r| \le \omega_{\max}, |u_1| \le u_{\max}, |u_2| \le u_{\max} \quad (47)$$

Equations (44) describe a kinematics of the vehicle. Equations (45) describe a dynamics of the vehicle.

The initial state $x_1(0)$, $x_2(0)$ of the vehicle belongs to some area Ω. Let the purpose state of the vehicle is given by $x_1=0$, $x_2=0$. The orientation of platform is arbitrary.

From section IV of this article we get

$$u_1 = u_{\max}\tanh\left(q^2(-\varphi_1 b_{11} - \varphi_2 b_{21})\right),$$
$$u_2 = u_{\max}\tanh\left(q^2(-\varphi_1 b_{12} - \varphi_2 b_{22})\right), \quad (48)$$

$$\varphi_1 = \omega_l - \omega_{\max}\tanh\left(q^1(-x_1\alpha_{11} - x_2\alpha_{21})\right),$$
$$\varphi_2 = \omega_r - \omega_{\max}\tanh\left(q^1(-x_1\alpha_{12} - x_2\alpha_{22})\right). \quad (49)$$

Let the Lyapunov function is given by:

$$V = 0.5\left(\varphi_1^2 + \varphi_2^2\right) \quad (50)$$

Differentiating the Lyapunov function (50) we obtain:

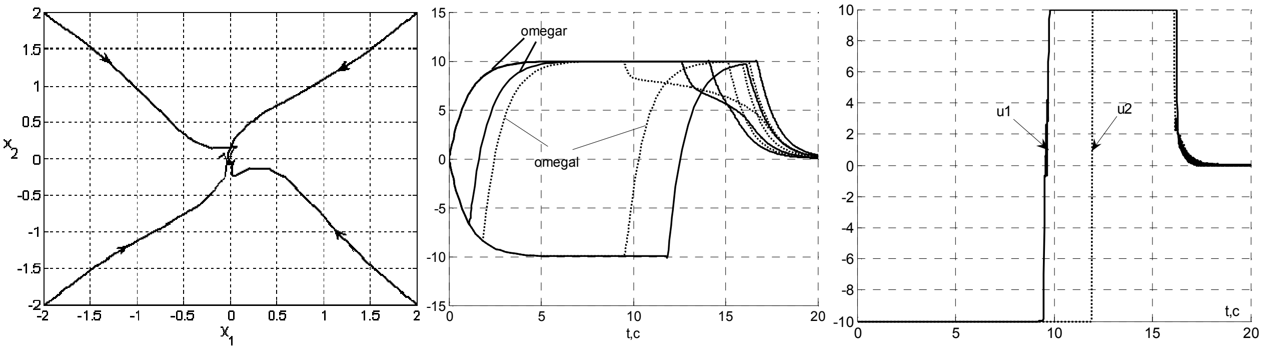
$$\dot{V} = \left[b_{11}u_{\max}\tanh q^2(\varphi_1 b_{11} + \varphi_2 b_{21}) + \right.$$
$$+ b_{12}u_{\max}\tanh q^2(\varphi_1 b_{12} + \varphi_2 b_{22}) + d_1\omega_l\left.\right]\varphi_1 +$$
$$+\left[b_{21}u_{\max}\tanh q^2(\varphi_1 b_{11} + \varphi_2 b_{21}) + \right. \quad (51)$$
$$+ b_{22}u_{\max}\tanh q^2(\varphi_1 b_{12} + \varphi_2 b_{22}) + d_2\omega_r\left.\right]\varphi_2$$

Function (51) is a negative definite function if

$$\left|b_{11}+b_{12}\right|u_{\max}>\left|d_1\omega_l\right|,\left|b_{21}+b_{22}\right|u_{\max}>\left|d_2\omega_r\right|. \tag{52}$$

$$\left|\alpha_{11}+\alpha_{12}\right|\omega_{\max}>0,\left|\alpha_{21}+\alpha_{22}\right|\omega_{\max}>0. \tag{53}$$

There are modeling results of system (44) – (49) in fig. 5, 6, and 7. Parameters of fig. 5, 6, and 7 modeling results are: $u_{\max}$=10, $\omega_{\max}=10$, r=0.2, a=1, $J\in[0.05\ 0.15]$, $d_i\in[0.5\ 3]$, $b_{11}=b_{22}=1$, $b_{12}=b_{21}=0$, $q^1=q^2=10$.



a) b) c)

Fig.5 – a) Path of mobile robot; b) Wheel's speeds of the mobile robot; c) Fig. 7. The control action of the mobile robot

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